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ON USAGE OF INDUCTIVE APPROACH IN MATHEMATICAL TEXTBOOKS AT SECONDARY SCHOOL

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Abstract
Quality of mathematical education is very actual didactic issue at present. One possibility how to increase it is a use of unconvencional experimentally inductive approach. The appropriate textbook then can be a useful helper for teacher who intends to apply this approach. There are given an analysis of the usage of inductive and deductive approach in mathematical textbooks here. The research was focused on the content of mathematics textbooks for secondary school, on their interpretative and exercise text components, respectively. The main interest was to explore the representation of inductive and deductive methods in interpretation of the mathematics textbooks and to measure the content of the exercises whose student's solution requires the use of heuristic approaches. The results of the analysis show that each author prefers a different approach to interpretation which is used in the writing of textbooks. Interpretative means most contained in all the textbooks are a direct interpretation with the motivation (34.4%), a direct interpretation without motivation (31%) and an indirect interpretation with the motivation (24.5%). Inductive and deductive interpretative means are only rarely represented in all the textbooks. The highest rate was registred in the case of task using interpretation with the motivation (7%).
Key words: *deduction, induction, mathematical education, mathematical textbook, secondary school.*

Introduction

Currently, many studies in the field of didactic of mathematics are devoted to issues related to the use of heuristic approaches to teaching mathematics at all levels of schools (see eg long-term research of the TIMSS, or Prince, J. M., Felder, R. M. (2006), Kopka, J. (2007)). These studies demonstrate the necessity of using the activating teaching and learning methods and their considerable positive impact on effectivity of education. The use of this approach for teaching the pupils evidently improves their learning, knowledge and skills. Consequently, pupils are able to better understand the genesis of new concepts, their inclusion into a logical

structure and the causes of their definition. These reasons lead to many recommendations to use heuristic approaches for teaching of mathematics much more than was practiced previously. Their presence can be useful to improve mastery of concepts and relations between them and a better understanding of the taught topic.

Despite of traditional deductive approach in mathematics most mathematical theories have both an experimental and inductive character. Their beginnings arise out of tentative searching and speculative trial and error; they gain a deductive character only after their period of investigation. Investigations, as described in Kopka, J. (2004), is a method of teaching and learning mathematics which permits students to enter and penetrate more deeply into the world of mathematics that most other teaching approaches fail to do. If one wishes the students to have experiences of how mathematics evolves, then it should be respected how mathematical theories come into existence, how they develop and how they finally gain their form and nature. Too frequently students are only exposed to mathematics in its final and approved form. Using investigations is one method of teaching involved in the full range of the development of a mathematical theory. Investigations also provide students with insights into what it is like to be a mathematician and to experience of mathematical thinking at work. Students should be able to investigate certain mathematical situations and consequently to formulate problems and hypotheses. This inductive approach should be completed by validation of the hypotheses, ie by return deduction. Clearly, the inductive way is much more time-consuming and difficult for teachers and students than traditional one. On the other hand, it contains very important and worth student activities which are all about making the students more active participants in the learning process – an observation, an investigation, formulation and solving of problems and formulation and validation of hypotheses.

Inductive Teaching and Learning Approaches

As mentioned in Emanovský, P., Břehovský, J. (2010) the most commonly used inductive teaching and learning methods are inquiry learning, problem-based learning, project-based learning, case-based teaching, discovery learning and just-in-time teaching (Prince, J. M., Felder, R. M. (2006)). The investigations according to Kopka, J. (2004) is possible to consider as a method of the first cathegory. The inquiry learning means that students are presented with questions to be answered, problems to be solved, or a set of observations to be explained (Bateman, W. (1990)). If the method is implemented effectively, the students should learn to „formulate good questions, identify and collect appropriate evidence, present results systematically, analyze and interpret results, formulate conclusions, and evaluace the worth and importace those conclusions“ (Lee, V. S. (2004)).

Textbook and its Functions

A textbook can be seen from several points of view – as an element of curricular project, as a part of a package of educational means, or as a part of a didactic text (Průcha (1998)). Commonly, a textbook is considered as a didactic text that its form allows learning and its content and scope embraces certain part of the curriculum. Each curriculum therefore sets out some way of designing textbooks and every textbook has its contents correspond with this document. The textbook is an integral part of the educational process and it has a direct impact to this process, as the apparatus of control not only pupils' learning, but also teachers' teaching. In this sense, a textbook performs several functions which are using by its users, whether students or teachers. In this view, one can distingush two basic function of textbooks:

Function of textbooks for students: textbboks are source from which pupils learn and acquire the knowledge, skills, values, etc (Průcha (1998)),

Function of textbooks for teachers: textbooks are the source from which teachers plan to use the content of the curriculum as a direct presentation of content in their own teaching (Průcha (1998)).

Both these features point to how and why the textbooks are used by different subjects, and thus what is the role of textbooks in the educational process. It is clear that the effect of textbooks on teaching and the results of teaching certainly cannot be underestimated just because so many teachers use their contents to create thematic plans, to prepare their interpretation and they also select the teaching method, exercises and homework for pupils described in the textbook. For these reasons it is possible to use the textbook to familiarize teachers with the heuristic approaches, and to some extent, influence the choice of teaching methods.

In contrast, using the textbooks students should acquire knowledge, skills and attitudes required by the curriculum in the form of key competencies. From the perspective of mathematics textbooks and in connection with the heuristic approaches is mainly the development of the following skills through which students acquire the required knowledge: to work with mathematical concepts, to apply mathematical knowledge in practice or other subjects, to discover and work creatively, to think logically, to prove propositions, to solve problems.

The aim of the analysis of textbooks was to explore the extent to which currently used mathematics textbooks for secondary schools fulfill the role of teachers' and pupils' guide to heuristic approaches. The primary analysis is focused on the actual state of the usage of inductive and deductive approaches in the textbooks and the actual state of the representation of problems requiring these approaches.

Research Problems

The research was focused on the content of mathematics textbooks for secondary schools and their interpretative and exercise text component, respectively. The main interest was to explore the representation of inductive and deductive methods in interpretation of the mathematics textbooks and to measure the content of the exercises whose student's solution require the usage of heuristic approaches. From didactic point of view it is very important to know whether given textbook is suitable for the development of key competencies which should be acquired by pupils in the educational process.

Currently, the most used comprehensive range of mathematics textbooks in secondary schools is Mathematics for colleges and secondary vocational schools 1 – 6 (Publishing House SPN) and Mathematics for secondary grammar schools (Publishing House Prometheus). For this reason, these two series of mathematics textbooks were the subject of the analysis. The objective was to answer the following questions:

- Do the authors of selected mathematics textbooks use the inductive and deductive approaches in their interpretation? If so, to what extent?
- Do the authors of selected mathematics textbooks enter exercises whose student's solution requires usage of heuristic approaches? If so, to what extent?
- If there are used inductive approaches in the interpretation and practice, what thematic groups are so used in?

Analysis of Textbooks

The aim of the analysis was to map the incidence of inductive and deductive approaches used in interpretation and tasks in the above mentioned mathematics textbooks. The research goal was not to complete the evaluation of these books, but only the recognition of the interpretative and training means in their frequency and type. The actual analysis of the textbooks has focused on two parts of books: interpretative part and training part. Analysis of the two parts is given separately because of clarity of the results.

Interpretative part of the Textbooks

There has been studied explanatory text including the use of motivation in this part of the analysis. The classification of the used interpretative means was created during the initial analysis of the text of all the textbooks, which then was used to map their abundance in the textbooks (direct interpretation, indirect interpretation, tasks using interpretation and heuristic strategies). Each group was subdivided into two subgroups (with motivation and without motivation). Characteristics of each group are shown in Table 1.

Table 1. List of structural components of textbook

Interpretative means	**Characteristic**
Direct interpretation	Author directly explains concepts and introduces new concepts, gives definitions, followed by practice.
Indirect interpretation	Author uses a solution of a problem to interpretation. The solution encounters a new problem, which is further defined, the author deals with the problems, does not give space to reader, derives formulas, etc.
Tasks using interpretation	Author enters a task to reader (to farmulate a definition, to draw a graph of the function, etc.). The active collaboration assumes a reader who has given tasks to accomplish and then continue reading the text. In the following may be mentioned the right solution and the set of all tasks leads to new knowledge. Author herself or himself summarizes everything.
Heuristic strategy	.Author requests a reader to solve a problem and requires an active collaboration with the reader, who is forced to look for a solution formulate the hypothesis and the subsequent verification. The author may lead reader to the correct solution.

The characteristics of the interpretative means show that the first two ones (direct and indirect interpretation) can not be regarded as an inductive or deductive approaches. Author

provides the reader purposefully no chance for discretion or own way of dealing only communicate certain information. The author aim is to provide the reader a source of knowledge which is more or less passively accepted by the reader. The following two means of interpretation (tasks using interpretation, heuristic strategies) is described more in detail.

The choice of the interpretative means "tasks using interpretation" directly assumes the reader's participation. The reader is forced to actively participate in solving problems, and thus acquires the necessary knowledge and skills. The readers must find solutions based on their previous experience and directly participate in the discovery of partial solutions of given problems. Final summary or verification of newly acquired knowledge is back in the hands of the author. This method can be considered as an inductive approach.

The last interpretative means "heuristic strategy" can be considered as an inductive approach as well. The problem is presented to reader who is forced to solve it. The reader must figure out a strategy of solving the problem, formulate hypotheses and then establish their validity. The author in this case provides the reader some guidance that helps him to solve the task. This method of interpretation is very difficult for the reader, but gives her or him a huge space for her or his own self-improvement and develops a comprehensive understanding of the issue. Moreover, all the right solutions that students invent have a great motivational effect. Heuristic strategies are typically used for tasks that are designed to link the component parts of the curriculum and to help the reader to create a comprehensive idea of the discussed topics (eg Odvárko, O., Calda, E. Kolouchová, J. Řepová J .: Mathematics for colleges and secondary vocational schools 6, SPN Praha 1987, Section: Hypotheses and their validation, p. 254). This approach can be described as inductive and deductive.

Further, it was shown for all registred interpretative means, if the reader is motivated by the author in some way. In all textbooks, the authors used various forms of motivation, from the practical lessons learned in mathematics and other subjects, over the real-world situations and ends with mapping the historical development of the concepts. There was made no distinction among the various forms of motivation. It was always just written, whether any form of motivation for the interpretative means was used or not.

The analysis consisted in determining the percentage of individual methods in these textbooks. The number of chapters of the book or its parts was taken as a base because of use different means of interpretation in the chapters. In some cases, the chapter could be divided for this reason to several separate parts. The following example is given for illustration:

The textbook Mathematics 3 was divided into 33 teaching sections (or chapters), where were identified the following educational resources in:

• direct interpretation without motivation: 22,
• indirect interpretation with motivation: 11,

$$d = \frac{22}{33} \cdot 100 = 67\% ,$$

$$im = \frac{11}{33} \cdot 100 = 33\% ,$$

thus the direct interpretation without motivation (*d*) was used in 67% and the indirect interpretation with motivation (*im*) was used in 33% of the interpretative means. The following Table 2 lists the particular percentage representation of the interpretative means used in the textbooks:

Table 2. The percentage incidence of interpretative means in particular textbooks

TEXTBOOK TITLE	INTERPRETATIVE MEANS							
	Direct Interpret. [%]		Indirect Interpret. [%]		Tasks using interpret. [%]		Heuristic strategy [%]	
	without motiv.	with motiv.	without motiv.	with motiv.	without motiv.	with motiv.	without motiv.	with motiv.
Mathematics for colleges and secondary vocational schools								
Mathematics 1	97,5	2,5	0	0	0	0	0	0
Mathematics 2	20	0	0	80	0	0	0	0
Mathematics 3	67	0	0	33	0	0	0	0
Mathematics 4	29	26	0	45	0	0	0	0
Mathematics 5	32	49	0	19	0	0	0	0
Mathematics 6	53	21	0	21	0	0	0	5
Mathematics for secondary grammar schools								
Sequences and series	0	7	0	14	29	50	0	0
Functions	0	25	0	30	0	45	0	0
Analytic Geometry	63	37	0	0	0	0	0	0
Differential and integral calculus	0	90	0	10	0	0	0	0
Complex numbers	0	85	0	15	0	0	0	0
Planimetry	24	52	24	0	0	0	0	0
Equations and Inequalities	10	17	0	73	0	0	0	0
Stereometry	0	88	0	12	0	0	0	0
Combinatorics, Probability, Statistics	0	82	0	18	0	0		0
Goniometry	0	24	0	18	0	53	0	5

For a more comprehensive overview of the use of interpretative means in all of these textbooks it was created the Table 3 and Graph 1 as result of the analysis. There is shown the percentage of the defined interpretative means in the interpretation of both series of mathematics textbooks for secondary schools in Table 3. To calculate the percentage of

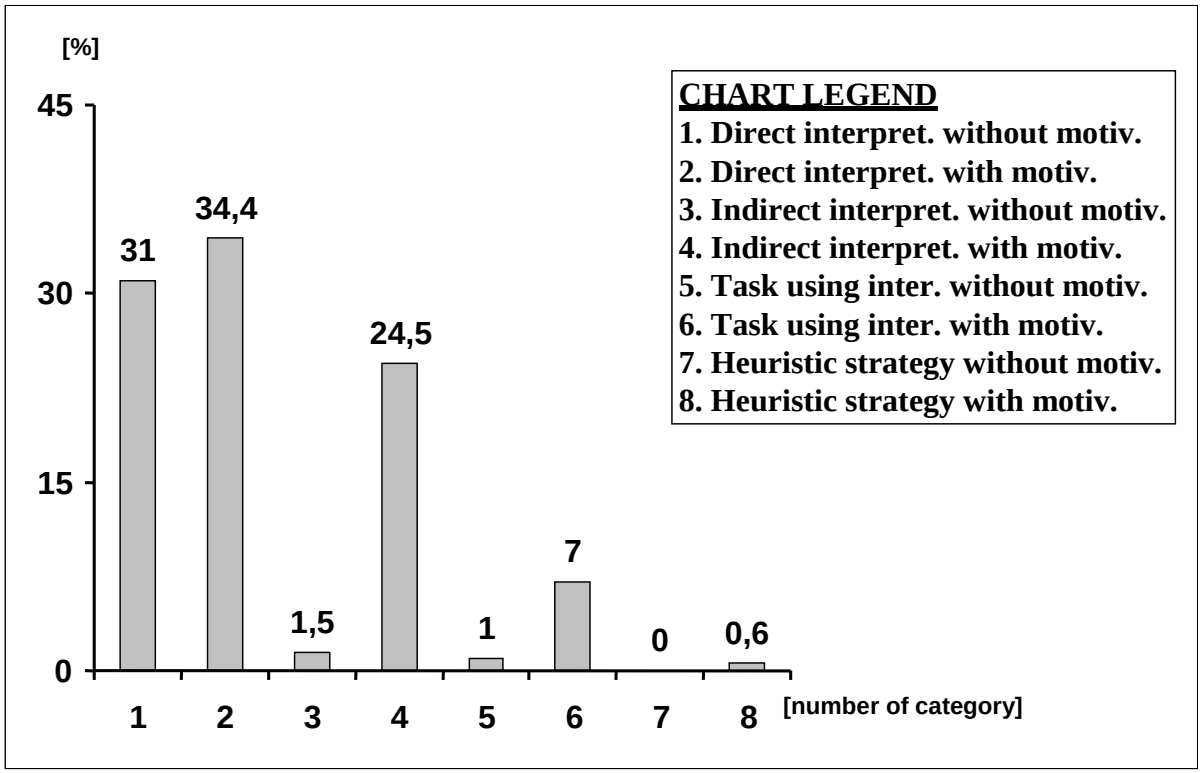
individual interpretative means there was used the similar calculation as in the previous illustrative example.

Table 3. The total percentage incidence of interpretative means in the textbooks

INTERPRETATIVE MEANS			
Direct interpretation [%]		Indirect interpretation [%]	
without motivation	with motivation	without motivation	with motivation
31	34,4	1,5	24,5
Tasks using interpretation [%]		Heuristic strategy [%]	
without motivation	with motivation	without motivation	with motivation
1	7	0	0,6

The following Graph 1 was generated by the Table 3. There is the percentage of the interpretative means in the interpretation of all textbooks translated into a clear graphical form here.

Graph 1. The total percentage incidence of interpretative means in the textbooks.



Training part of the Textbooks

Considering the large number of examples in all the textbooks, we first made their individual categories. The set of all the examples used in textbooks was divided into three categories: question, calculation, inductive and deductive problems.

Table 4. Characteristic of the categories of the examples

Category name	Characteristic	Example of task
question	These are jobs that require a direct answer. There is no need to count.	Explain the concept of domain of function
calculation	Solver uses known calculation to solve problems, only familiarities are practiced.	Calculate the first five members of an arithmetic sequence, if you know: $a_1 = 2$, $d = 3$
inductive and deductive problems	The reader must look for process of solutions, to formulate the hypothesis, to verify them, to verify or prove submitted or discovered claims.	Prove the theorem. Explore the divisibility of numbers $n^2 + 5n + 6$, where n is an integer. Formulate a hypothesis. (p. 258: Odvárko, Calda, Kolouśková, Řepová,: Mathematics 6, SPN 1987.)

The analysis of this part of the textbooks focused on types of examples that are used in the exercises. It directly related to the frequency of finding problems that require the solver to use inductive and deductive approach for solutions in the sense of the categories characterized in the Table 4. To get an overview of the use of this type of examples in the textbooks, we calculated the percentage of inductive and deductive problems in the textbooks.

There were not distinguished the ceses of only inductive or only deductive character of examples. The investigated examples could have only inductive character (eg derivation of a formula, formulation of a hypothesis), or only deductive character (eg prove that the formula is true) or a combination of both approaches.

The following example is given for illustration:

The textbook Sequences and Series:

• the number of problems: 136,
• the number of inductive and deductive problems: 26,

$$p = \frac{26}{136} \cdot 100 = 19{,}1\% \,,$$

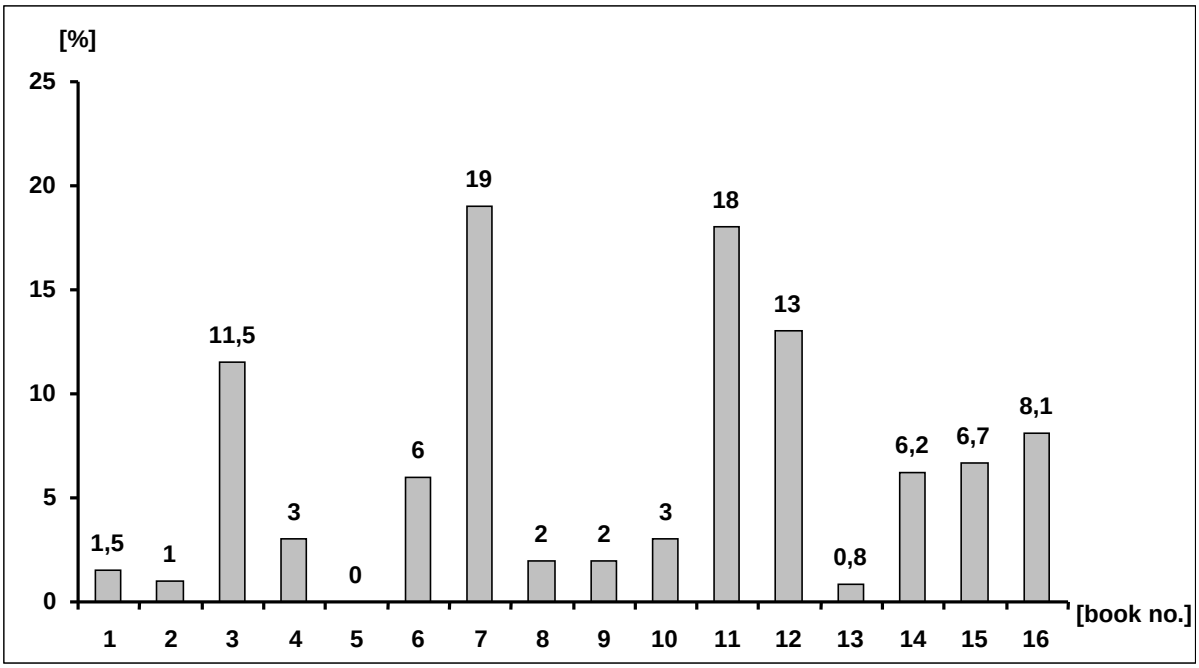
thus the percentage representation of the inductive and deductive problems (*p*) in the textbook Sequences and Series is 19,1%. The following Table 5 lists the particular percentage representation of the inductive and deductive problems used in the textbooks:

Table 5. The percentage incidence of inductive and deductive problems in the textbooks

Number of textbook	Textbook title	Percentage of inductive and deductive problems
1	Mathematics for colleges and secondary vocational schools 1	1,5
2	Mathematics for colleges and secondary vocational schools 2	1
3	Mathematics for colleges and secondary vocational schools 3	11,5
4	Mathematics for colleges and secondary vocational schools 4	3
5	Mathematics for colleges and secondary vocational schools 5	0
6	Mathematics for colleges and secondary vocational schools 6	6
7	Sequences and Series	19
8	Functions	2
9	Analytic geometry	2
10	Differential and integral calculus	3
11	Complex numbers	18
12	Planimetry	13
13	Equations and Inequalities	0,8
14	Stereometry	6,2
15	Combinatorics, Probability, Statistics	6,7
16	Goniometry	8,1

The following Graph 2 was generated by the Table 5. There is the percentage of the incidence of inductive and deductive problems in all textbooks translated into a clear graphical form here.

Graph 2. The percentage incidence of inductive and deductive problems in the textbooks



Discussion

The results of the analysis show that each author prefers a different approach to interpretation, which is used in the writing of textbooks. The following interpretative means there were most represented in all textbooks: direct interpretation with motivation (34.4%), direct interpretation without motivation (31%) and indirect interpretation with motivation (24.5%). Inductive and deductive interpretation means are only sparsely represented in all textbooks. The highest rate was registered in the case of „task using interpretation with motivation“ (7%). The textbooks, which inductive and deductive interpretative means are used in most are: Mathematics for colleges and secondary vocational schools 6, Goniometry, Functions and Sequences and Series. One can find also interpretation with the heuristic strategy in these books. It can be assume that use of the inductive and deductive interpretation means in the textbooks is appropriate and has a significant benefit for the reader. Textbooks with the highest representation of inductive and deductive tasks are: Sequences and Series (19%), Complex numbers (18%) and Planimetry (13%). In the case of textbooks for secondary technical and vocational schools has the largest representation of these tasks Mathematics for Secondary School Vocational 3 (11.5%), which includes the following thematic sections: Functions, Goniometry and Trigonometry and Stereometry. Again, we can conclude that the use of these tasks in the textbooks is suitable and has a great contribution to understanding the curriculum and the overall connection of acquired knowledge.

The results of the described research correspond with several published analyses which conclude that inquiry-based teaching and teaching with heuristic strategy is generally more effective than traditional instruction for achieving a variety of learning outcomes (Smith, D. (1996), Haury, D. (1993), Shymansky, J., Hedges, L., Woodworth, G. (1990), Hohn, L., Frey, B. (2002)).

Conslusions

Overall, the outcome of the analysis of textbooks shows that the representation of the inductive and deductive approaches described above used in the textbooks is quite uncommon. If we aim to allow teachers and pupils to use heuristic strategies in much greater extent than is currently, it is necessary to offer them a much larger number of manuals, examples and the possibilities to use inductive and deductive approaches in the textbooks. From this point of view, we opine that the representation of heuristic approaches in mathematics textbooks for secondary schools is insufficient.

A natural question arises whether the increase in the proportion of those methods would help improvement of the process of teaching and learning. We believe that greater representation of inductive and deductive methods would affect the quality of teaching and at the same time, helped to increase the use of these methods by teachers themselves. A serious response to these assumptions could be given by a further research.

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